Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL - 2018

M.Sc. Mathematics

16PMTCC19 – GRAPH THEORY

Duration of Exam – 3 hrs ***Semester – IV*** ***Max. Marks – 70***

Part A (5X2= 10 marks)

Answer **ALL** questions

1. Define: Graph isomorphism.
2. Define: Maximal non-Hamiltonian graph.
3. Define: Vertex Connectivity.
4. Define: Jordan curve.
5. Define: Map coloring.

Part B (5X5 = 25 marks)

Answer **ALL** questions

6a. Define: Simple graph with example. Also state and prove First theorem of graph theory.

OR

6b. Prove that: For any given two vertices *u* and *v* in a graph *G*. Every *u*-*v* walk contains a path.

7a. Define: Cycle with example. Also prove that: Let *G* be a graph with degree of every vertex is at least two then graph *G* contains a cycle.

OR

7b. Prove that: A connected graph *G* has an Euler trail if and only if it has either no vertex of odd degree or exactly two vertices of odd degree.

8a. Define: Bridge with example. Also prove that: Let *G* be a connected graph then *G* is a tree if and only if every edge *e* of *G* is a bridge.

OR

8b. Define: Tree with example.
Also prove that for a tree T with at least one edge and $P = u_0, u_1, u_2, ..., u_n$ be a path of maximum length n in T. Then $d(u_0) = 1 = d(u_n)$.

9a. Define: Planar graph with example.
Also prove that: Let *G* be a simple planar graph then *G* has a vertex of degree less than six.

OR

9b. State and prove Kuratowski’s theorem.

10a. Prove that: Let *G* be a nonempty graph. Then $\chi(G) = 2$ if and only if *G* is a bipartite graph.

OR

10b. Define: Critical graph with example.
Also prove that: Let *G* be a *k*-critical graph then degree of every vertex in *G* is at least (*k-1*).

Part C ***(5X7 = 35 marks)***

Answer **ALL** questions

11a. State and prove characterization of Bipartite graph.

OR

11b. State and prove characterization of disconnected graph.

12a. Prove that: the following statements are equivalent for a connected graph *G*.

i) Graph *G* is an Eulerian.

ii) Every vertex of *G* has even degree.

iii) The set of edges of *G* can be partitioned into cycles.

OR

12b. State and prove characterization of Hamiltonian graph.

13a. Prove that : For a simple graph *G*, if *u* and *v* are two distinct vertices of *G* then graph *G* is a tree if and only if there is precisely one path from *u* to *v*.

OR

13b. Prove that: Let *e* be an edge of a graph *G* and *G*-*e* be the sub graph of *G* obtained by removal of an edge *e* from *G*. Then $w(G) \leq w(G-e) \leq w(G)+1$.

14a. State and prove Euler’s formula.

OR

14b. Prove that: Let *G* be a simple planar graph with *n*-vertices and *e*-edges, where $n \geq 3$. Then $e \leq 3n-6$.

15a. Define: Edge colouring of a graph *G*.

Prove that: If $G = K_n$ is a complete graph with n vertices with $n \geq 2$. Then

$$\text{¡}_1(G) = \begin{cases} \Delta(G) & \text{,if n is even} \\ \Delta(G)+1 & \text{,if n is odd} \end{cases}.$$

OR

15b. Define: Chromatic number of a graph *G*

Also prove that: For any graph *G*, $\text{¡}(G) \leq \Delta(G)+1$.